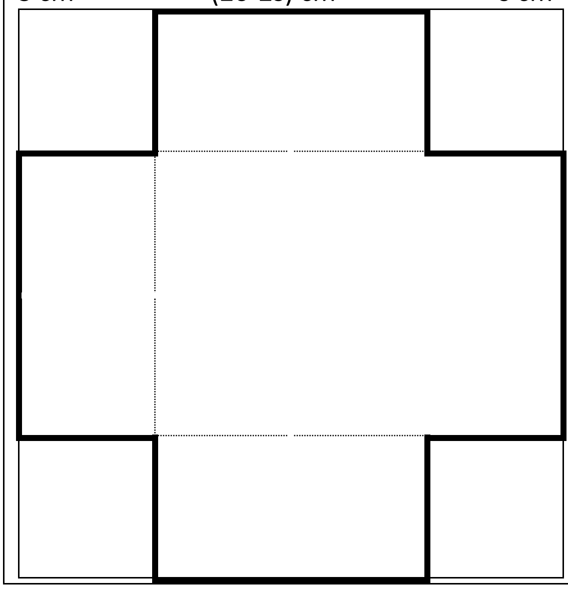
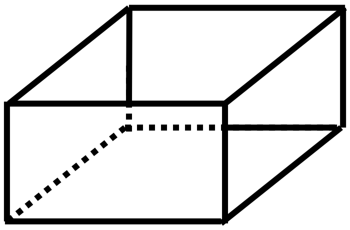
What is the maximum volume of an open topped box made from a 20cm x 20cm sheet of cardboard?

When squares measuring scm x scm are cut from the corners the box will have a base measuring

(20 – 2s) cm by (20 – 2s) cm and a height of s cm.

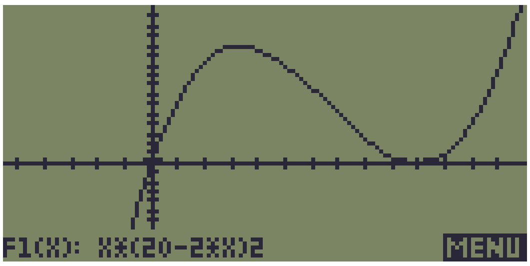
S cm (20-2s) cm s cm



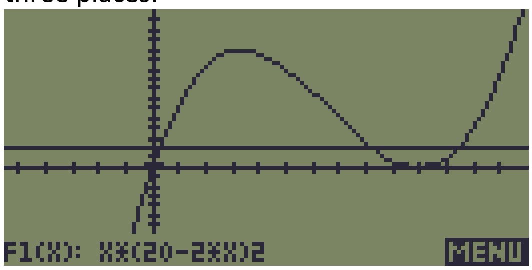


This box has volume $V = s(20 - 2s)^2$

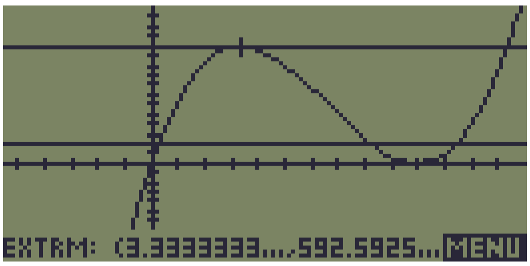
The graph of V against s is the cubic shown below:



A horizontal line will sometimes cut the cubic in three places:



If the horizontal line corresponds to a maximum (or minimum) value of the volume then it will still cut the curve at three places but two of them will be identical:



The values of s which make practical sense are those between 0 and 10.

What happens when you cut squares of edge 0 or 10 from the 20cm x 20cm sheet of paper?

Using your graph, what is the maximum volume? What is the value of s that gives the maximum volume?

Using algebra

The maximum value of the volume and the corresponding value of s can be confirmed algebraically.

Let the horizontal line have equation $V = m$

Solving it simultaneously with $V = s(20 - 2s)^2$

gives $s(20 - 2s)^2$ = *m* where the values of *s* give the edge lengths of the squares which make the box have volume *m*.

Expanding the left hand side and collecting like terms gives:

$$s(400 - 80s + 4s^2) - m = 0$$

$$4s^3 - 80s^2 + 400s - m = 0$$

For finding the maximum or minimum, we know that two of the values of will be identical so the factorized form of the left hand side will be: $4(s - a)(s - b)^2$ which when expanded is:

$$4s^3 - (4a + 8b)s^2 + (8ab + 4b^2)s - 4ab^2$$

From this: $4a + 8b = 80$, $8ab + 4b^2 = 400$ and the maximum and minimum volumes are $m = 4ab^2$

For the maximum and minimum volumes we are interested in the value of *b* so *a* should be eliminated from the these equations to give $a = 20 - 2b$

and hence $2b(20 - 2b) + 2b^2 = 100$

Expanding and collecting like terms gives $40b - 4b^2 + b^2 - 100 = 0$ or $3b^2 - 40b + 100 = 0$

which can be factorized to give $(3b - 10)(b - 10) = 0$

The solutions are $b = 3\frac{1}{3}$ and $b = 10$

The second solution corresponds to cutting squares of edge 10cm yielding no volume.

The first gives the maximum volume as $\frac{10}{3}\left(20 - \frac{20}{3}\right)^2 = \frac{10}{3} \times \left(\frac{40}{3}\right)^2 = \frac{16000}{27}$ = 592. 592…..cm^3

Group Activity

As a group activity, after completing the 20x20cm activity, each student could use a different square and then a general result could be deduced and even proven.

As a class activity, a general result for beginning with a rectangle could be investigated.