

SMC 2019 SAMPLE PROBLEMS

PRIMARY QUESTIONS

1. 5 is 20% of what number?
2. What is the value of $1 + 2 - 3 + 4 - \dots - 99 + 100$?
3. How many numbers between 0 and 2019 are perfect squares?
4. Write as a simple fraction $(\frac{1}{2}) / (\frac{1}{4} + \frac{1}{5})$
5. A rod of length 64 cm is divided into three pieces in the ratio 2 : 4 : 6
What is the length of the longest piece?
6. Jack is 3 times as old as Jill. In 7 years time, he will be twice as old as Jill.
How old is Jack?
7. What is the average if 7 and $\frac{1}{7}$?
8. Find two numbers whose sum is 56 and such that one number is three times as large as the other?
9. By how much does $\frac{1}{3}$ of $2\frac{1}{2}$ exceed $\frac{1}{2}$ of $\frac{1}{5}$?
10. What is the area of the triangle ABC where A is the point (-2,3), B is point (7,4) and point C is at (11,2)?
11. Harper is $\frac{2}{3}$ of the way through her hike. After walking another 500 m, she is $\frac{3}{4}$ of the way through her hike. What is the total length of her hike?
12. How many numbers between 1 and 1000 are divisible by either 7 or 11?
13. A, B and C are three different digits such that
$$A B C + A C B = B C A$$
Find the value of A, B and C.
Note : ABC is a three digit number
14. A, B and C are digits such that the number ABC is divisible by 3, the number BAC is divisible by 4 and the number BCA by 5. The number CBA has an odd number of factors.
Find the number ABC

JUNIOR PROBLEMS

1. What is the area of a triangle with sides 16, 17 and 17 ?
2. A rod of length 190 cm is cut into three pieces whose lengths are proportional to 3, 5 and 11 What is the length of the shorter piece?
3. What is the value of $\frac{2}{3} - (\frac{1}{5} - \frac{1}{4})$?
4. How many 2 digit numbers, from 10 to 99, have the sum of their digits divisible by 6?
5. A copper block in the shape of a rectangular solid measures 2 x 3 x 4. It is melted down and made into 3 equal cubes. What are the dimensions of these cubes?

6. The perimeter of a rectangle is 28. A second rectangle is 3 times as long and twice as wide. The perimeter of the second rectangle is 72, What is the area of the first rectangle ?
7. Pam is $\frac{2}{3}$ of the way along her walk to school. After walking a further 300 m she is $\frac{3}{4}$ of the way to school, How far is her walk to school?
8. How many positive integers between 1 and 100 are divisible by
 - i) 2, 3 and 7?
 - ii) either 2, 3 or 7?
9. What is the smallest number such that its distance from - 2 is equal to twice its distance from 5?
10. $N = 1 \times 3 \times 5 \times 7 \times \dots \times 2017 \times 2019$ What is the last digit of N?
11. Samantha has 200 \$1 coins She wishes to give the coins away to her friends in such a way that each friend gets at least one coin and no two people get the same number of coins.
What is the largest number of friends that she can give coins away to ?
12. A gambler lose $\frac{2}{3}$ of his stake in the first hour of gambling but wins back $\frac{4}{5}$ of his losses from the first hour in the second hour. What fraction of his original stake does he now have left?
13. The measure of each interior angle of a regular polygon is 15 times the measure of each exterior angle. How many sides does the polygon have ?
14. How many integers from 0 to 2019 end in 1 ?
15. A ,B and C are digits such that $BC + ABC + ABC = 876$
What is the value of $A + B + C$?
16. A rope is cut in half and one half is thrown away. Then $\frac{1}{5}$ of the remaining half is cut off and used to tie a parcel, The piece left is 12 m long,
How long was the rope originally?
17. Find the sum of the first 100 terms of the sequence
1, -2, 3, -4, 5, -6, 7, ...
18. How many 5 digit numbers can you make using two 3's, 2 5's and one 9?
19. What is the value of $4^3 \cdot 8^5$ as a power of 2?
20. If $\frac{2}{x} = \frac{3}{y} = \frac{x}{y}$ what values can $x + y$ take?

INTERMEDIATE PROBLEMS

1. What is the value of $\frac{4}{5} - (\frac{1}{7} - \frac{1}{8})$?
2. What is the area of a triangle with sides 10, 12 and 12?
3. A rhombus ABCD has sides of length 5. Given A is point (3,3) , AD has slope $\frac{1}{2}$ and AB has slope 2 what are the coordinates of C?
4. Consider the number 13335555777777799999999911 ...113131313...

- What is the 1000th. digit in this number?
5. The first two integers for which $1 + 2 + 3 + 4 + \dots + n$ is a perfect square are 1 and 8. What are the next two numbers for which this is true?
 6. If $2a = 3b + 5$ what is the value of $\frac{4^a}{8^b}$?
 7. The product of two or more consecutive positive integers is 47AB74 where A and B are digits. Find these consecutive integers?
 8. What is the sum of the digits of the smallest positive integer which is divisible by 99 and has all its digits equal to 2 ?
 9. Triangle ABC has $AB = 8$, $AC = 9$, $BC = 7$. O is the incentre of the triangle D is on AB, E is on AC such that DE passes through O and is parallel to BC. Find the length of DE?
 10. The lengths of the edges of 3 cubes are all integers. If the sum of their surface areas is 564 cm^2 , what are the possible values of their volumes?
 11. In triangle ABC, angle $BAC = 60^\circ$, X lies $\frac{1}{3}$ of the way from B to C, and AX bisects angle BAC. Find the size of angle ABC?
 12. In triangle ABC E is the midpoint of BC and F is the midpoint of EC. D is on AB such that $BD : DA = 1 : 5$. If the area of the triangle ABC is 18 cm^2 what is the area of the quadrilateral AFED ?
 13. Find the value of n for which $98! \times 9900 = n!$
 14. I have twenty 3cent stamps and twenty 5 cent stamps . Using one or more stamps how many different amounts of stamps can I make ? Answer 152
 15. Find the integer n such that $3^2 \times 3^5 \times 3^8 \times \dots \times 3^{3n-1} = 3^{442}$.

SENIOR PROBLEMS

1. Solve the equation $|x|(|x| - 5) = -6$
2. The line $y = 3x + 4$ intersects the parabola $y = x^2 + x$ in two points A and B, Find the length AB.
3. Find all the ordered pairs (p,q) for which $p! = q^2 - 12$
4. Solve the inequality $\frac{1}{x} + 2x > 3$
5. The function f satisfies the equations
 $f(0) = 0$, $f(2n) = (n)$, $f(2n + 1) = f(n) + 1$.
 What is the value of $f(2019)$?
6. The triangle ABC is right angled at A. $AC = 3$, $AB = 1$ and the angle bisector of angle B meets AC at P. Find CP ?
7. A sequence is formed by taking the products of the corresponding terms of two arithmetic sequences. The first three terms are 10, 17 and 86. What is the 11th term?
8. Find the values of a, b and c given that

$$a^2 + 1 = 3b + c$$

$$b^2 + 33 = 7c - 3a$$

$$c^2 - 5 = b - 3a$$

9. A rectangular solid with sides 2, 10 and 22 is inscribed in a sphere of radius r . Find the value of r .
10. How many 9-letter strings consisting entirely of the letters A and B do not contain the consecutive letters ABBA? Answer 338
11. What is the side length of the equilateral triangle ABC for which there is a point P inside the triangle such that $AP = 6$, $BP = 8$ and $CP = 10$?
12. Two fair dice, one red and one white, are rolled. What is the probability that the number on the red die is as large as that on the white die?
Ans 7/12
13. A_1, A_2, \dots, A_n are consecutive vertices of a regular n -gon such that

$$\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$$

What is the value of n ?

14. How many lattice points lie on the graph of $y = (x - 9)^{9-x^2}$?
15. A spotlight is placed at the point $(-9,0)$ and is pointed towards a hill whose equation is given by $y = -x^2 - 8x - 7$. Find the smallest angle the spotlight should be pointed so that the light beam is visible on the other side of the hill? Answer; $\arctan 2$
16. Sketch the graph of $f(x) = |x| + |2x - 1| - |3x - 2|$
Find the area inside the square $-1 \leq x \leq 1$, $-1 \leq y \leq 1$ and above the graph of $f(x)$
17. Find p such that the roots of $5x^2 - 5px + 66p = 1$ are positive integers?
Answer 76
18. If $0 \leq A \leq 2\pi$, what is the maximum value of $\sin A (1 + \cos 2A)$?
19. An 8 sided die, a 12 sided die and a 20 sided die are rolled. Find the probability that one of the values rolled is equal to the sum of the other two values rolled? Answer 23/240
19. Two positive numbers a and b satisfy the equations
 $2 + \log_2 a = 3 + \log_3 b = \log_4 (a+b)$
What is the value of $\frac{1}{a} + \frac{1}{b}$?
20. In triangle ABC, point K divides BC such that $BK : KC = 1 : 5$. point L is on AC such that $AL : LC = 2 : 5$. O is the point of intersection of AK and BL. Find the ratio $BO : OL$.