Example of Mathematical Modelling

The following is designed to explain the processes of mathematical modelling as it is an important form of mathematical inquiry and highlights how mathematical modelling can be used to support the teaching of mathematics.

This unit, and the accompanying unit, “How animals keep their cool”, explore how mathematics can be used to solve problems and make suggestions for possible project topics.

The Australian Curriculum Problem Solving Proficiency states

- Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. Students formulate and solve problems when they use mathematics to represent unfamiliar or meaningful situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable.

This particular unit relates to the Year 10 Content Description:

Using units of measurement
Solve problems involving surface area and volume for a range of prisms, cylinders and composite solids (ACMMG242)

BABY IN THE CAR INVESTIGATION[1]

It was a February heatwave. Michael Jones was driving to the shops with his six-month old son. He parked his car, grabbed his shopping list, and looked at his son who was now asleep. He thought to himself that he would only be about twenty minutes so he decided to leave him in the car. He wound up all of the windows, locked the doors and went shopping.

A short time later, on returning to the car he saw a woman smash his side window. He confronted the woman and accused her of attempting to steal his son.

The woman said that all she was doing was saving his son’s life.

Why did the woman think that the baby’s life was endangered? Was it possible? If so would Michael Jones be safe if he was sitting in the car in similar circumstances?

To tackle a real-world problem like this we use modelling. The modelling process involves:

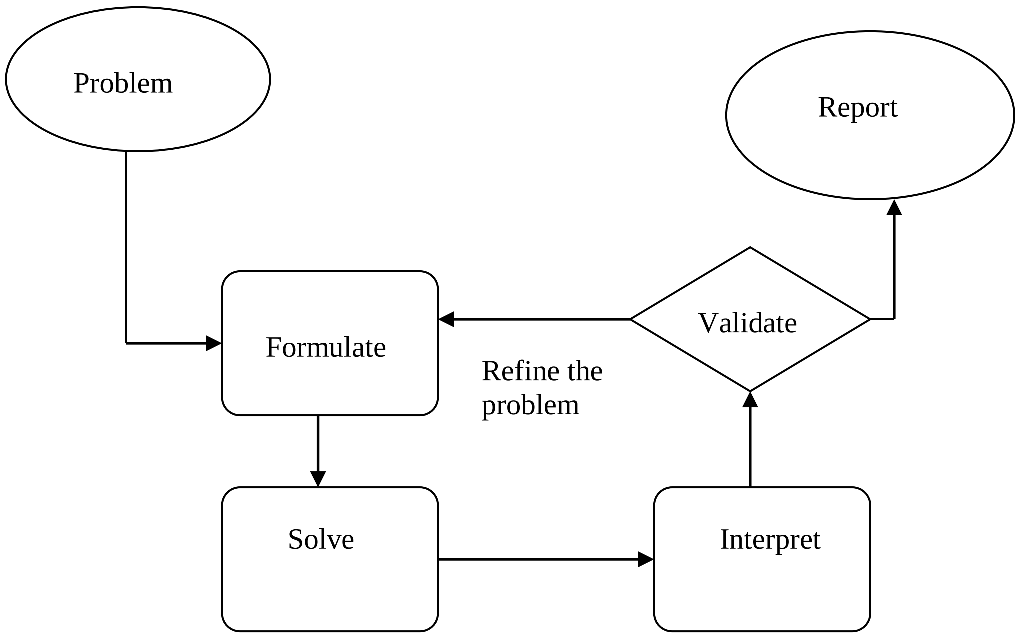
- **Problem**: Taking a real-world problem and clarifying the problem so you know what to look for

[1] Adapted from a Mathematics Curriculum and Teaching Program (MCTP) Unit

- **Formulate**: formulating a mathematical model which mirrors some aspect of the problem. It will be necessary to choose the most appropriate method to solve the problem.
- **Solve**: solving the model by using mathematical tools to find a possible answer to the mathematical problem
- **Interpret**: Interpreting the solution by working out what your mathematical answer means when you look back at the original problem
- **Validate**: check whether your solution is useful, review the model for improvement and, if necessary, repeating the cycle
- **Report**: explain what you have done and your conclusions

Problem solving strategies may include:

- making a model, picture or diagram
- looking for a pattern
- guessing and checking
- making assumptions
- creating an organised list
- making a table or chart
- solving a simpler problem
- working backwards
- using logical reasoning



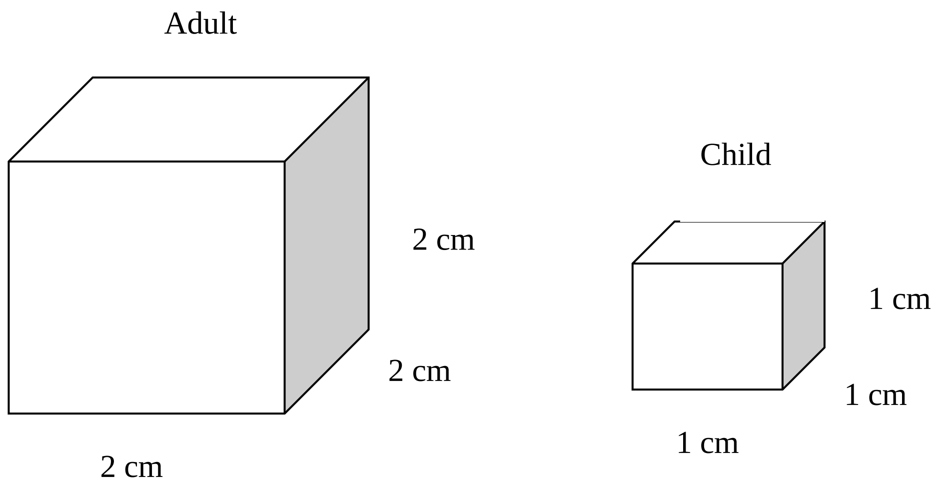
Clarifying the problem

Some questions to be addressed:

- What does the problem tell you?
- What does the problem ask you?

- List possible reason’s for the woman’s actions? (This may include kidnapping, dehydration, baking etc)
- Why did Michael Jones assume it was safe to leave the baby in the car?
- Discuss each reason and select the most likely explanation
- Can the problem be restated and simplified where appropriate?
- Formulate an hypothesis
- What is dehydration? How is water lost from the body?

Formulate – building a simple model



Solve

In the model, how many surfaces are there through which water can evaporate? What are the respective volumes and surface areas?

For the adult:			For the child:		
Surface Area (SA)	=	6 x 2 x 2	Surface Area (SA)	=	6 x 1 x 1

	=	$24\ cm^2$		=	$6\ cm^2$
Volume (V)	=	2 x 2 x 2	Volume (V)	=	1 x 1 x 1
	=	$8\ cm^3$		=	$1\ cm^3$

	Surface Area (SA) cm^2	Volume (V) cm^3	Surface Area/Volume (SA/V)
Baby	6	1	6
Adult	24	8	3

Interpreting the model

The ‘baby’ has twice as much surface area per unit of volume available for evaporation as an ‘adult’.

Validate

These calculations indicate that a small cube has a large surface area relative to its volume (a large surface area per unit volume) while a larger cube has a smaller surface area relative to its volume (a smaller surface area per unit volume). This may explain why the baby would dehydrate much more quickly than an adult. However, using a cube as a model is a very poor approximation for a person or baby. Do we get the same results if we use a better approximation?

Refining the model

The model above is a very simple one. Could it be improved to more realistically represent a baby and an adult? Does the principle still hold?

For example:

- Assume the body is in the shape of a cylinder with the diameter being one-sixth of the height. Calculate the surface area and volume for different heights. (Plotting the graph of SA/V against the height may be useful).

- Use the ambulance driver’s ‘rule of thumb’ for surface area (the surface area of the body is approximately 100 times the surface area of a hand) and calculate approximate volumes from weight and density.

Report

A completed report should include:

- an introduction that outlines the problem situation to be explored, including its significance, its features, and the context
- the method of solution in terms of the mathematical model or strategy to be used
- the appropriate application of the mathematical model or strategy, including:

– the generation or collection of relevant data and/or information, including details of the process of collection
– mathematical calculations and results, and appropriate representations (e. g. tables, graphs)_
– the analysis and interpretation of results
– reference to the limitations of the original problem, as well as appropriate refinements and/or extensions

▪ a statement of the solution and outcome in the context of the original problem.

Try These

1 The common ant needs about 24 mL of oxygen for every cubic centimetre of its body. Having no lungs, the ant absorbs the oxygen through its 'skin' at a rate of about 6.2 mL per second per square centimetre of skin.

In the movie 'Them', we are presented with gargantuan ants 8 metres long and 1.5 metres in radius.

Would these creatures survive?

(Treat the ants as cylinders)

2 Most reptiles and amphibians are small, cold-blooded creatures, which means they do try to sustain an internal body temperature different from their surroundings.

From what we have discussed, give a probable reason for this.

Investigations

3 Investigate the relationship between volume and height of the human body, assuming a body is the same shape as a cylinder, with the diameter one-sixth of the height.

- Calculate the volume for heights 60, 90, 120, 150 and 180 cm.
- Plot the graph of volume against height, comment on the strengths and weaknesses of the model.
- Suggest ways in which the model might be improved

By calculating the surface area, complete the following table.

Height (*h*)	0	60	90	150	180	210
Volume (*V*)						
Surface Area (*SA*)						
$\frac{SA}{V}$						

- Plot the graph of $\frac{SA}{V}$ against *h* and discuss what the implications are for the human body as it grows.

Each year in Australia, unfortunate situations occur when babies die from dehydration. Sometimes they were left asleep in cars while their parents did some ‘quick’ shopping.

- Why is it that babies dehydrate more quickly than adults?

(Humans, and other warm-blooded animals, experience heat control through the pores of their skin. When they perspire the body cools down)

4 Why is it not possible for humans to grow double their present height without some structural change?

5 Obtain a copy of Gulliver’s Travels by Jonathon Swift and investigate the information given about Gulliver’s dimensions in comparison to the people of Lilliput indicates whether Swift knew anything about the changes in surface area and volume compared to the changes in height.

6 Why…

do elephants and rhinos have big ears and wrinkled skin?
do smaller animals have more fir than most larger animals?

do smaller animals tend to be shaped more like a sphere than larger animals?