2018 SMC SAMPLE QUESTIONS for Junior / Intermediate / Senior

Sample Questions for **Juniors**

1. Find all the 3-digit multiples of 7 for which the sum of the digits is also a multiple of 7.

2. Joe walks up10 steps going up either 1 or 2 steps at a time. There is a snake on the 6th step so he cannot stop on this step. In how many different ways can Joe reach the top step?

3. How many pairs (x,y), where x and y are whole numbers , satisfy the condition that 2x + 7y = 100?

4. By how much does $\frac{5}{7}$ of $9\frac{1}{3}$ exceed $\frac{3}{11}$ of $4\frac{2}{5}$?

5. N is a whole number and S(N) is the sum of the digits of N.
   Thus, for example, if N = 372 then S(N) = 12
   If N = 1982 then S(N) = 20
   Find all the numbers for which N + S(N) = 2010.

6. 90% of the members in a certain gym club are girls and there are 3 boys in the club. How many members are there in the club?

7. Amanda has received a 5% raise. Now she earns $12,000 more than her friend Lisa. Before Amanda's salary raise, Lisa's salary was 1% higher than Amanda's. What is Lisa's salary?

8. i) How many 4-digit palindromic numbers start with the digit 7?
   ii) How many palindromic numbers lie between 2000 and 20,000?

9. The sum of the 3-digit numbers 35x and 4y7, where x and y are different   digits, is N.
   i)   What is the smallest value that N can take?
   ii)   What is the largest value that N can take?
   iii)   If N is divisible by 36 what is the value (or values) that N can have?

10. Consider the sequence of numbers
    1 , 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, …
    If this pattern continues what will be
    i)   the 2018th number in the sequence,
    ii)   the sum of the first 2018 terms in the sequence?

1. Jill walks up 12 steps going up either 1 or 2 steps at a each stride. There is a snake on the 7th step so she cannot stop there. In how many different ways can Jill reach the top step?

2. A chemist has 100 cc of a liquid which contains 20% alcohol and 80% water. She adds more alcohol to make a solution with $33\frac{1}{3}$% alcohol. How much water must she now add in order to return it to a 20% alcohol solution?

3. How many 3-letter sequences can be made using the letters in the word "ADELAIDE" ? (For example, "LAA" and "ALA" are acceptable but "AAA" is not.)

4. N is a 6-digit number formed using an arrangement of the digits 1, 2, 3, 3, 4, 5. What is the i)   smallest N,   ii)   largest   N that is divisible by 66?

5. Peta solves the equation ax – b = c for x and Rosa solves the equation bx – c = a for x. If they both get the same answer for x and a, b, and c are both different and non-zero, what is the relation between a, b and c?

6. The lengths of the equal sides of an isosceles triangle are (x + 1) and the length of the third side is (3x + 2). Find all the possible values of x.
(The triangle should not be degenerate – become a straight line --and your answer need not be an integer and could be an inequality.)

7. What is the number of positive integers n for which there is a triangle with 3 acute angles and the side lengths are 10, 24 and n?

8. The numbers 1, 2, 3, 4, …., 100 are divided into two groups A and B. You take a number from A (call it a) and a number from B (call it B) and you add them to make the number (a + b). What is the largest value of (a + b) that you can make?
How many different answers you can make for (a + b)?

9. ABCD is a rhombus with sides of length 1. O is a point on diagonal AC such that AD = AO = x and OD = OB = OC. Find the value of x.

10. N = 9,999,999   How many 9's in the number N^2 ?

11. Let S = { 15, 24, 30, 40, 50, 60, 80 } When one number is removed, the product of the remaining numbers is a perfect cube. What is this perfect cube?

12. Alex puts 20 green marbles in a hat, Lynn puts 16 purple marbles in the hat and Josh places k blue marbles into the hat. If a marble is randomly selected from the hat, the probability that it will be blue is 1/5 . What is the value of k ?

13. A rectangle has an area of 120 cm^2 and a diagonal of length 17 cm. What is the perimeter of the rectangle ?

14. Find all the positive prime numbers p such that $p^{2018} + p^{2019}$ is a perfect square.

1. What reduced fraction $\frac{a}{b}$ with $4 < b < 15$ is closest to $\frac{3}{7}$?

2. Solve the system of equations $\quad x^2 + x\sqrt[3]{xy^2} = 208$

   and $\quad y^2 + y\sqrt[3]{yx^2} = 1053$.

3. Two arithmetic sequences are multiplied together term by term to form another sequence whose first three terms are 468, 462 and 384. What is the fourth term?

4. Let p be a prime number. If p years ago, the ages of three children formed a geometric sequence with a sum of p and a common ratio of 2, what is the sum of the children's ages now?

5. If $x + \frac{1}{y} = 12$ and $y + \frac{1}{x} = \frac{3}{8}$ find all the solutions for x and y.

6. What is the value of $\sqrt{2000.2008.2009.2017 + 1296}$?

7. In the triangle ABC, AB = 4, BC = 6 and AC = 8. Squares ABQR and BCST are drawn external to and lie in the same plane as triangle ABC. Find the length QT.

8. Consider the sequence 1, 8, 15, 22, 29, 36, 43, … where each number is obtained from the previous one by adding 7.
   a. If T_n = 7n + k, what is the value of k?
   b. T_1 = 1 and T_6 = 36 are square numbers. Find the next three square numbers.
   c. Find a rule for finding the square numbers in the sequence, i.e. if T_n is a square number what is the rule for finding k in terms of n?

9. Find the smallest positive integer n such that the fractions in this sequence
$$\frac{3}{n+5} , \frac{4}{n+6} , \frac{5}{n+7} , \dots , \frac{13}{n+5}$$
are all in lowest form.

10. The roots of $ax^2 + bx + c = 0$ are 6 and p. The roots of $cx^2 + bx + a = 0$ are q and r. Find the product pqr ?

11. Find the smallest prime p larger than 10, which can be expressed in the form
$\quad p = x^3 - y^3$ where x and y are positive integers.

12. Triangle ABC is right angled at B. D lies on CB such that AD bisects the angle CAB and CB = 8 DB. Find the value of sin(angle CAD)?

1. 14   2. 24   3. 6   4. 82/15   5. 2004 , 1986   6. 27g 3b
7. 303,000   8. a. 9000 b. 280   9. i) 758 ii) 855 iii) 792, 828
10. i) 2 ii) 5043

1. 11x5 = 55   2. 40 cc   3. 5040   4. i) 123354 ii) 533412   5. a+b+c = 0
6. -2/3 < x < 0   7. 3 (23.24.25)   8. largest 199 : number 98
9.   10. 6   11. $2^{12}3^{3}5^{6}$ (60 removed)
12. 9   13. 14   14. p = 3

1. 5/12   2. x = 8, y = 27   3. 234   4. 28 (p = 7)
5. x = 8, y = ¼   6. 4,034,036 = $(2008.2009 - 36)^{2}$ 7. 8
8, a. k = -6   b. 1, 35, 64, 169, 225, 400, 484, 729, 842, …
            c, and these terms are 1, 6, 10, 25, 33, 58, 70, 105, 121, …
9. n = 17   10. ac/6   11. 19   12. $\dfrac{\sqrt{21}}{7}$

All care taken with both questions and answers. Good luck